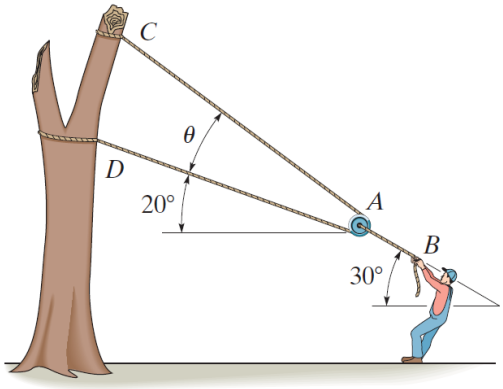
Problem 3-7

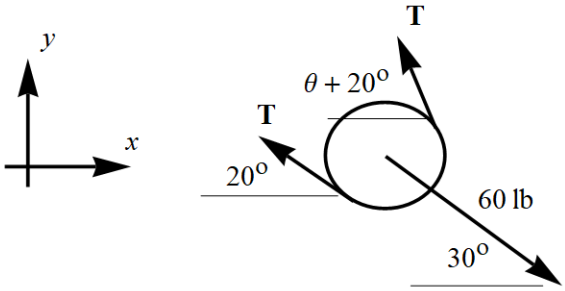
The man attempts to pull down the tree using the cable and *small* pulley arrangement shown. If the tension in AB is 60 lb, determine the tension in cable CAD and the angle θ which the cable makes at the pulley.



Prob. 3–7

Solution

Draw a free-body diagram for the pulley, noting that because it's small (and frictionless) the tension is the same everywhere in the cable.



In order for the pulley to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad 60\cos 30^\circ - T\cos 20^\circ - T\cos(\theta + 20^\circ) = 0 \tag{1}$$

$$\sum F_y = 0: \qquad -60\sin 30^\circ + T\sin 20^\circ + T\sin(\theta + 20^\circ) = 0 \tag{2}$$

Solve equation (1) for T

$$T = \frac{60\cos 30^\circ}{\cos 20^\circ + \cos(\theta + 20^\circ)}$$

and plug it into equation (2).

$$-60\sin 30^\circ + \frac{60\cos 30^\circ}{\cos 20^\circ + \cos(\theta + 20^\circ)}[\sin 20^\circ + \sin(\theta + 20^\circ)] = 0$$

Solve this equation for θ.

$$\frac{\sin 20^\circ + \sin(\theta + 20^\circ)}{\cos 20^\circ + \cos(\theta + 20^\circ)} = \tan 30^\circ$$

$$\sin 20^\circ + \sin(\theta + 20^\circ) = \tan 30^\circ[\cos 20^\circ + \cos(\theta + 20^\circ)]$$

$$\sin 20^\circ + \sqrt{1 - \cos^2(\theta + 20^\circ)} = \tan 30^\circ \cos 20^\circ + \tan 30^\circ \cos(\theta + 20^\circ)$$

$$\sqrt{1 - \cos^2(\theta + 20^\circ)} = (\tan 30^\circ \cos 20^\circ - \sin 20^\circ) + \tan 30^\circ \cos(\theta + 20^\circ)$$

Square both sides.

$$1 - \cos^2(\theta + 20^\circ) = (\tan 30^\circ \cos 20^\circ - \sin 20^\circ)^2 + 2(\tan 30^\circ \cos 20^\circ - \sin 20^\circ) \tan 30^\circ \cos(\theta + 20^\circ) + \tan^2 30^\circ \cos^2(\theta + 20^\circ)$$

$$(\tan^2 30^\circ + 1) \cos^2(\theta + 20^\circ) + 2(\tan 30^\circ \cos 20^\circ - \sin 20^\circ) \tan 30^\circ \cos(\theta + 20^\circ) + [(\tan 30^\circ \cos 20^\circ - \sin 20^\circ)^2 - 1] = 0$$

$$\cos(\theta + 20^\circ) = \frac{-2(\tan 30^\circ \cos 20^\circ - \sin 20^\circ) \tan 30^\circ \pm \sqrt{[2(\tan 30^\circ \cos 20^\circ - \sin 20^\circ) \tan 30^\circ]^2 - 4(\tan^2 30^\circ + 1)[(\tan 30^\circ \cos 20^\circ - \sin 20^\circ)^2 - 1]}}{2(\tan^2 30^\circ + 1)}$$

$$\cos(\theta + 20^\circ) \approx \{-0.940, 0.766\}$$

$$\theta + 20^\circ = \{160^\circ, 40^\circ\}$$

$$\theta = \{140^\circ, 20^\circ\}$$

Since $0^\circ < \theta < 90^\circ$, we choose $\theta = 20^\circ$. Therefore,

$$T = \frac{60 \cos 30^\circ}{\cos 20^\circ + \cos(\theta + 20^\circ)} \approx 30.5 \text{ lb}.$$